Capillary bonding

Vincent Richefeu and Farhang Radjai

Laboratoire de Mécanique et Génie Civil, CNRS - Université Montpellier 2, Place Eugène Bataillon, 34095 Montpellier cedex 05

1 Capillary cohesion

We present here the capillary cohesion resulting from a liquid bridge between two particles. The bridge between two particles of different sizes takes a complex shape as illustrated in figure 1. R_1 and R_2 are the particle radii, ζ_1 and ζ_2 are the filling angles (corresponding to the wetted surface of the particles), θ is the wetting angle, and δ_n is the distance between particles. The axis x coincides with the axis of rotation of the liquid bridge, and the coordinate y describes the profile of the meridian of the bridge as a function of x. The radius of the bridge is denoted y_0. The coordinates x of the three-phase contact line (also called triple line), i.e. the line defining the solid-liquid-gas interface, are denoted by x_{c1} and x_{c2} respectively for particles 1 and 2.

In the following, we assume that

- the particles are spherical;
- the particle surface is smooth so that the surface roughness is ignored;
- the water content is sufficiently low so that water is present in the form unconnected liquid bridges.
- the gravity effects are neglected so that the liquid bridges are not deformed under gravity;
- the capillary forces are studied in quasistatic regime so that the viscosity of the liquid can be neglected.

The pressure difference $\Delta p = p_{gaz} - p_{liquide}$ (also called capillary suction) through the liquid-gas interface is related to the curvature of the liquid bridge and the surface tension of the liquid σ by the Young-Laplace equation (Hotta et al. [1974], Soulié et al. [2006]):

$$\Delta p = \sigma \left[\frac{y''(x)}{\left(1 + y'^2(x)\right)^{3/2}} - \frac{1}{y(x)\sqrt{1 + y'^2(x)}} \right] \tag{1}$$

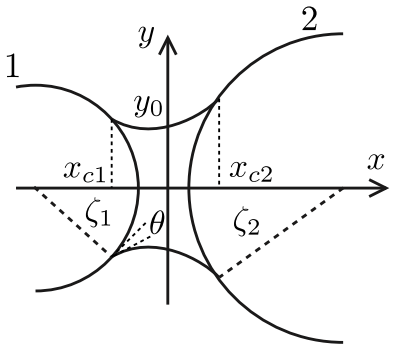

Fig. 1. Geometry of a liquid bridge between two particles of different sizes.

The volume of the bridge is given by:

$$V = \pi \int_{x_{c1}}^{x_{c2}} y^2(x)\,dx - \tfrac{1}{3}\pi R_1^3 (1 - cos\zeta_1)^2(2 + cos\zeta_1) - \tfrac{1}{3}\pi R_2^3 (1 - cos\zeta_2)^2(2 + cos\zeta_2) \tag{2}$$

The interparticle distance δ_n can be expressed in the coordinates x_{c1} and x_{c2}:

$$D = R_2 (1 - cos\zeta_2) + x_{c2} + R_1 (1 - cos\zeta_1) - x_{c1} \tag{3}$$

The capillary force can be calculated at the gorge (Hotta et al. [1974], Lian et al. [1993], Mikami et al. [1998]):

$$F = 2\,\pi\, y_0\, \sigma + \pi\, y_0^2\, \Delta p \tag{4}$$

The relationship between the capillary force and the shape of the liquid bridge is described by a system of nonlinear equations (1) to (4). However, in discrete numerical simulations an explicit expression of f_n is needed as a function of interparticle distance δ_n and volume of liquid bridge V for any set of parameters R_1, R_2, σ and θ.

2 Implementation

Numerical resolutions of the system of equations have been proposed by several authors (Soulie et al. [2006], Soulié et al. [2006], Scholtes et al. [2009b,a]). The major drawback of these approaches is that they do not provide a physically interpretable form of the capillary law. Moreover, they are not very efficient in terms of computation time. Richefeu it et al. (Richefeu et al. [2006], Richefeu

et al.) proposed a simple expression for the capillary force (figure 2b):

$$f_n = \begin{cases} -\kappa\sqrt{R_1 R_2} & \text{for } d_n < 0 \\ -\kappa\sqrt{R_1 R_2}e^{-d_n/\lambda} & \text{for } 0 \leq d_n \leq d_{rupture} \\ 0 & \text{for } d_n > d_{rupture} \end{cases} \tag{5}$$

where $\kappa = 2\pi\sigma\cos\theta$ (Willett et al. [2000], Bocquet et al. [2002], Herminghaus [2005]), and $\lambda = \lambda(V, R_1, R_2)$ expresses a length scale that governs the exponential decay of the capillary force as a function of the gap (Richefeu et al.). This explicit expression gives an excellent approximation of the capillary force obtained by the integration of the Young-Laplace equation; see figure 2b.

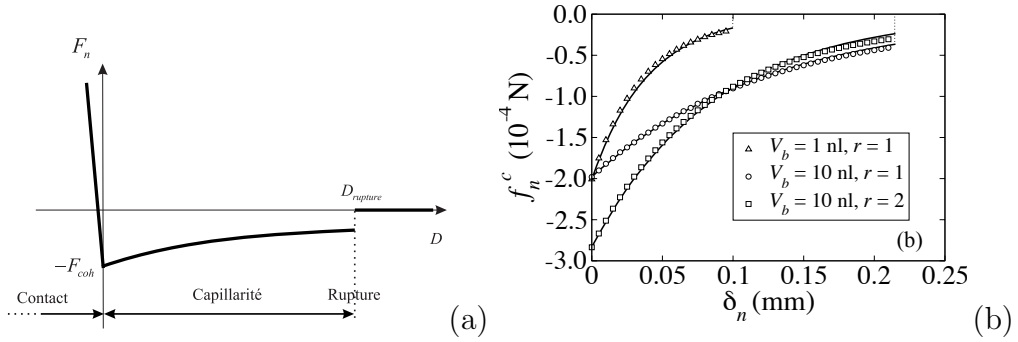

Fig. 2. (a) Representation of the interaction law in the case of a capillary bridge for a given volume of liquid bridge. (b) Capillary force between two particles as a function of δ_n. The calculation is performed for three values of the volume of the bridge and the reduced radius r (solid line) and compared to the prediction of the Young-Laplace equation (dotted line).

To complete this expression, a failure criterion of the liquid bridge is required. This failure is governed by energy considerations. It corresponds to a minimal liquid-gas surface interface (Pepin et al. [2000]). Using a numerical solution of the Young-Laplace equation, Erle *et al.* (Erle et al. [1971]) and De Bisschop and Rigole (Bisschop and Rigole [1982]) have proposed two very close solutions for the failure distance. They proposed an empirical criterion based on the filling angle and the radius of the gorge to evaluate the distance of separation. From similar considerations, Lian *et al.* (Lian et al. [1993]) proposed the following relationship between the debonding distance D_{rupt}, the volume of liquid bridge V and the wetting angle θ:

$$d_{rupture} = (1 + \frac{\theta}{2})\, V^{1/3} \tag{6}$$

This failure criterion has been used in various numerical studies (Richefeu et al. [2006], Soulie et al. [2006]). Notice that the debonding distance is not the same as the distance at which the liquid bridge reforms. This hysteresis

phenomenon has been discussed in the literature (Pepin et al. [2001], Soulié [2005]) and can be taken into account. A very common solution is to neglect the volume of the adsorbed liquid to the particle surface so that rejoining occurs at contact.

The distribution of liquid bridges in a granular medium is poorly studied (Fournier et al. [2005], Kohonen et al. [2004]). It plays an important role in the force transmission (Richefeu et al. [2009], Radjai and Richefeu [2009]) and in hydro-texturing phenomena (Rondet et al. [2009b,a]). The study of the distribution of the liquid is all the more difficult that the samples are polydisperse and that the configuration is three-dimensional. In the case of polydisperse media, the volume of all liquid bridges cannot be the same. A common way to allocate the water to the grains is to attribute to each liquid bridge a volume of liquid corresponding to a fraction of the total volume of liquid. This distribution cannot be consistent with the thermodynamic equilibrium between gas, solid and liquid phases since it implies a constant pressure in the liquid phase (the gas phase is percolating). For this reason, it is necessary to take into account the capillary suction in the formulation of local laws of cohesion.

Finally, it is sometimes necessary to take into account the viscosity of the liquid. The normal component f_n^d of the viscous force can be obtained from (Adams and Perchard [1985]):

$$f_n^d = 6\pi\mu R^\star \dot{\delta}_n \frac{R^\star}{\delta_n} \tag{7}$$

where μ is the dynamic viscosity of the liquid and $R^\star$ is the reduced radius. For the tangential component f_t^d, it is possible to use the following approximation (Goldman et al. [1967]) which is valid for small values of δ_n and for small filling angles ζ_1 and ζ_2:

$$f_t^d = (\frac{8}{15}\ln\frac{R^\star}{\delta_n} + 0.9588)\cdot 6\pi R^\star \dot{\delta}_t. \tag{8}$$

References

M. J. Adams and V. Perchard. The cohesive forces between particles with interstitial liquid. *Inst. Chem. Engng Symp.*, 91:147–160, 1985.

F. R. E. De Bisschop and W. J. L. Rigole. A physical model for liquid capillary bridges between adsorptive solid spheres: the nodoid of plateau. *Journal of Colloid and Interface Science*, 88:117–128, 1982.

L. Bocquet, E. Charlaix, and F. Restagno. Physics of humid granular media. *C. R. Physique*, 3:207–215, 2002.

M. A. Erle, D. C. Dyson, and N. R. Morrow. Liquid bridges between cylinders, in a torus, and between spheres. *AIChE Journal*, 17:115–121, 1971.

Z. Fournier, D. Gerimichalos, S. Herminghaus, M.M. Kohonen, F. Mugele, M. Scheel, M. Schulz, B. Schulz, C. Schier, R. Seemann, and A. Shudelny. Mechanical properties of wet granular materials. *Applied Physics: Condensed Matter*, 17:S477–S502, 2005.

A. J. Goldman, R. G. Cox, and H. Brenner. Slow viscous motion of a sphere parallel to a plane wall – i. motion through a quiescent fluid. *Chem. Engng. Sci.*, 22:637–651, 1967.

S. Herminghaus. Dynamics of wet granular matter. *Adv. Phys.*, 54:221–261, 2005.

K. Hotta, K. Takeda, and K. Iionya. The capillary binding force of a liquid bridge. *Powder Technology*, 10:231–242, 1974.

M.M. Kohonen, D. Geromichalos, M. Scheel, C. Schier, and S. Herminghaus. On capillary bridges in wet granular materials. *Physica A*, 339:7–15, 2004.

G. Lian, C. Thornton, and M. J. Adams. A theoretical study of the liquid bridge force between rigid spherical bodies. *Journal of Colloid and Interface Science*, 161:138–147, 1993.

T. Mikami, H. Kamiya, and M. Horio. Numerical simulation of cohesive powder behavior in fluidized bed. *Chemical Engineering Science*, 53(10):1927–1940, 1998.

X. Pepin, D. Rossetti, S. M. Iveson, and S. J. R. Simons. Modeling the evolution and rupture of pendular liquid bridges in the presence of large wetting hysteresis. *Journal of Colloid and Interface Science*, 232:289–297, 2000.

X. Pepin, S. J. R. Simons, S. Blanchon, D. Rossetti, and G. Couarraze. Hardness of moist agglomerates in relation to interparticle friction, granule liquid content and nature. *Powder Technology*, 117:123–138, 2001.

F. Radjai and V. Richefeu. Bond anisotropy and cohesion of wet granular materials. *Philosophical Transactions of the Royal Society A-mathematical Physical and Engineering Sciences*, 367(1909):5123–5138, 2009.

V. Richefeu, M.S. El Youssoufi, R. Peyroux, and F. Radjai. A model of capillary cohesion for numerical simulations of 3d polydisperse granular media. *Int. J. Numer. Anal. Meth. Geomech.*, 32.

V. Richefeu, F. Radjai, and M.S. El Youssoufi. Stress transmission in wet granular materials. *Eur. Phys. J. E*, 21:359–369, 2006.

V. Richefeu, M. S. Youssoufi, E. Azema, and F. Radjai. Force transmission in dry and wet granular media. *Powder Technology*, 190(1-2):258–263, 2009.

E. Rondet, M. Delalonde, T. Ruiz, and J. P. Desfours. Identification of granular compactness during the kneading of a humidified cohesive powder. *Powder Technology*, 191(1-2):7–12, 2009a.

E. Rondet, M. Rundgsiyopas, T. Ruiz, M. Delalonde, and J. P. Desfours. Hydrotextural description of an unsaturated humid granular media: Application for kneading, packing and drying operations. *Kona-powder and Particle*, 27:174–185, 2009b.

L. Scholtes, B. Chareyre, F. Nicot, and F. Darve. Micromechanics of granular materials with capillary effects. *International Journal of Engineering Science*, 47(1):64–75, 2009a.

L. Scholtes, P. Y. Hicher, F. Nicot, B. Chareyre, and F. Darve. On the capillary stress tensor in wet granular materials. *International Journal For Numerical and Analytical Methods In Geomechanics*, 33(10):1289–1313, 2009b.

F. Soulié. *Cohésion par capillarité et comportement mécanique de milieux granulaires*. PhD thesis, Université Montpellier 2, 2005.

F. Soulié, F. Cherblanc, M.S. El Youssoufi, and C. Saix. Influence of liquid bridges on the mechanical behaviour of polydisperse granular materials. *International Journal for Numerical and Analytical Methods in Geomechanics*, 30(3):213–228, 2006. URL `http://dx.doi.org/10.1002/nag.476`.

F. Soulie, M.S. El Youssoufi, F. Cherblanc, and C. Saix. Capillary cohesion and mechanical strength of polydisperse granular materials. *Eur. Phys. J. E*, 21:349–357, 2006.

C. Willett, M. Adans, S. Johnson, and J. Seville. Capillary bridges between two spherical bodies. *Langmuir*, 16:9396–9405, 2000.